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Influence of the protective strip properties on distribution of the temperature at transient frictional heating

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ABSTRACT

The analytical solution of a boundary-value problem of heat conduction for tribosystem, consisting of the homogeneous semi-space, sliding uniformly on a surface of the strip deposited on a semi-infinite substrate, is obtained. For materials of frictional systems: steel–aluminum–steel and steel–zirconium dioxide–steel, the evolution and distribution on depth from a surface of friction for temperatures and thermal fluxes, is studied.

1. Introduction

Protective strips such as evaporated coatings and films are used for improvement of wear-contact characteristics of friction elements. It is known that addition of aluminum in frictional materials leads to higher and stable values of friction coefficient than for steel without aluminum. One of the most perspective materials used in heat-shielding coatings is the zirconium dioxide ZrO_2 based ceramics [1].

In the previous investigations authors examined the influence of the coating's physical properties on distribution of temperature and thermal stresses in the substrate. The two versions of statements of the corresponding thermal problems of friction, were used. In the first one, while calculating the temperatures, the intensity of the frictional heat flux, which has been directed to each components of friction pair, was assumed to be proportional to the specific capacity of friction [2,3]:

$$q = fVp_0. \quad (1)$$

In such statement at V = const. (the uniform sliding), the solution of a heat conduction problem of friction for the substrate with the homogeneous or composite strip on its surface, has been obtained [4–6]. The corresponding problem at $V(t) = V_0(1 - t/t_s)$, $0 \leqslant t \leqslant t_s$ (braking with constant retardation) has been studied in article [7,8]. The processes connected with the thermal cracking due to frictional heating on a surface of solid, consisting of semi-infinite substrate and a strip, have been considered in papers [9,10].

Other variant of statements of heat conduction problems of friction assumes the simultaneous solution of the equations of heat conduction and thermoelasticity for each of frictional elements with the subsequent calculation of heat fluxes intensities [11]. The solution of a transient heat conduction problem of friction for two semi-spaces in such statement has been obtained in article [12] and for tribosystem consisting of a strip sliding with constant speed on a surface of the semi-infinite substrate in article [13].

The statement and methods of the solution of heat conduction problem of friction are close to such cases when the problems of heat conduction are connected with mathematical modeling of the temperature fields in solids caused by the laser irradiation. In such problems it is supposed that heat flux intensity is uniform or is described by the linear function of time (the triangular time shape pulse) [14–16].

The main objective of the present article is to obtain the analytical solution of a thermal problem of friction for tribosystem consisting of three bodies: the homogeneous semi-space, sliding uniformly on a surface of the strip deposited on a semi-infinite substrate.

2. Problem formulation

The problem of contact interaction of two semi-spaces is considered, where one of them is homogeneous and the other is a semi-infinite substrate with surface covered by a strip of thickness d. The perfect heat contact between the strip and the substrate takes place. It is supposed, that the constant compressive pressures p_0 in direction of z axis of the Cartesian system of coordinates $Oxyz$ are applied to the infinities in semi-spaces (Fig. 1).

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Nomenclature

d	thickness of the strip
$\mathrm{erf}(x)$	Gauss error function
$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$	complementary error function
$\mathrm{ierfc}(x) = \pi^{-1/2}\exp(-x^2) - x\mathrm{erfc}(x)$	integral of the error function $\mathrm{erfc}(x)$
f	frictional coefficient
K	coefficient of heat conduction
k	coefficient of thermal diffusivity
p_0	pressure
$q = fVp_0$	intensity of the frictional heat flux (the friction power)
T	temperature
$T_0 = qd/K_s$	temperature scaling factor
$T^* = T/T_0$	dimensionless temperature
t	time
t_s	time braking
V	sliding speed
V_0	initial sliding speed
z	spatial coordinate

Greek symbols

$\tau = k_s t/d^2$	dimensionless time (Fourier's number);
$\zeta = z/d$	dimensionless coordinate

Indexes

f	bottom semi-space
s	strip
t	upper semi-space

The homogeneous upper semi-space slides with the constant velocity V in the direction of the y-axis on the strip surface. Due to friction the heat is generated on a contact plane $z = 0$. It is assumed that sum of the intensities of the frictional heat fluxes directed into each component of friction pair is equal with the specific friction power. Let us find the distribution of temperature fields and intensities of the heat fluxes in the frictional elements. Further, all values and the parameters concerning a top semi-space, strip and substrate will have bottom indexes "t", "s" and "f", respectively (Fig. 1).

The transient temperature fields $T_{t,s,f}(z,t)$ can be found from the solution of the following transient heat conduction problem of friction:

$$\frac{\partial^2 T_t(z,t)}{\partial z^2} = \frac{1}{k_t}\frac{\partial T_t(z,t)}{\partial t}, \quad 0 < z < \infty, \; t > 0, \tag{2}$$

$$\frac{\partial^2 T_s(z,t)}{\partial z^2} = \frac{1}{k_s}\frac{\partial T_s(z,t)}{\partial t}, \quad -d < z < 0, \; t > 0, \tag{3}$$

$$\frac{\partial^2 T_f(z,t)}{\partial z^2} = \frac{1}{k_f}\frac{\partial T_f(z,t)}{\partial t}, \quad -\infty < z < -d, \; t > 0, \tag{4}$$

$$T_s(0,t) = T_t(0,t), \quad t > 0, \tag{5}$$

$$K_s\frac{\partial T_s}{\partial z}\bigg|_{z=0-} - K_t\frac{\partial T_t}{\partial z}\bigg|_{z=0+} = q, \quad t > 0, \tag{6}$$

$$T_s(-d,t) = T_f(-d,t), \quad t > 0, \tag{7}$$

$$K_s\frac{\partial T_s}{\partial z}\bigg|_{z=-d+} = K_f\frac{\partial T_f}{\partial z}\bigg|_{z=-d-}, \quad t > 0, \tag{8}$$

$$T_t(z,t) \to 0, \quad z \to \infty, \; t > 0, \tag{9}$$

$$T_f(z,t) \to 0, \quad z \to -\infty, \; t > 0, \tag{10}$$

$$T_t(z,0) = 0, \quad 0 \leqslant z < \infty, \tag{11}$$

$$T_s(z,0) = 0, \quad -d \leqslant z \leqslant 0, \tag{12}$$

$$T_f(z,0) = 0, \quad -\infty < z \leqslant -d. \tag{13}$$

Let us denote by

$$\zeta = \frac{z}{d}, \quad \tau = \frac{k_s t}{d^2}, \quad K_f^* = \frac{K_f}{K_s}, \quad K_t^* = \frac{K_t}{K_s}, \quad k_f^* = \frac{k_f}{k_s}, \quad k_t^* = \frac{k_t}{k_s}, \tag{14}$$

$$T_0 = \frac{qd}{K_s}, \quad T_t^* = \frac{T_t}{T_0}, \quad T_s^* = \frac{T_s}{T_0}, \quad T_f^* = \frac{T_f}{T_0}. \tag{15}$$

Taking denotes (14) and (15) into account, the boundary-value problem of heat conduction (2)–(13) can be written down in the form

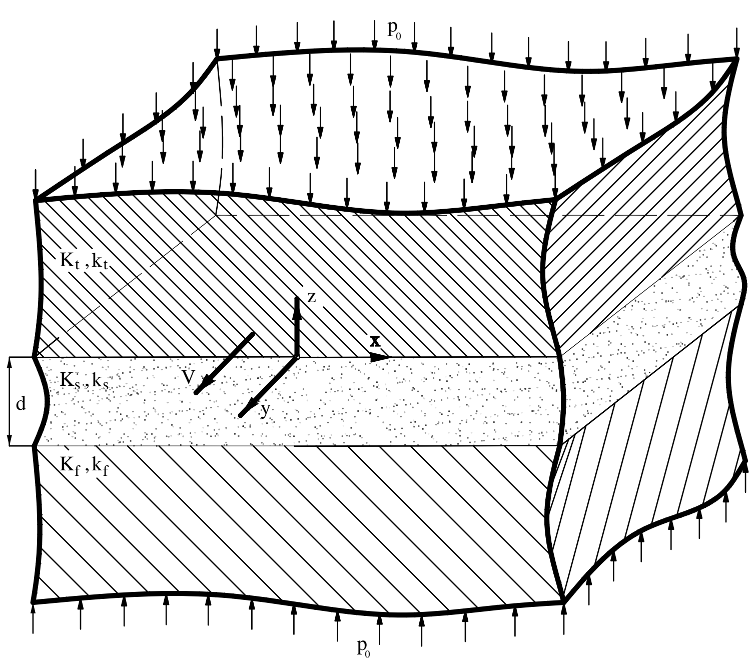

Fig. 1. Scheme of the problem.

$$\frac{\partial^2 T_t^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k_t^*}\frac{\partial T_t^*(\zeta,\tau)}{\partial \tau}, \quad 0 < \zeta < \infty, \; \tau > 0, \tag{16}$$

$$\frac{\partial^2 T_s^*(\zeta,\tau)}{\partial \zeta^2} = \frac{\partial T_s^*(\zeta,\tau)}{\partial \tau}, \quad -1 < \zeta < 0, \; \tau > 0, \tag{17}$$

$$\frac{\partial^2 T_f^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k_f^*}\frac{\partial T_f^*(\zeta,\tau)}{\partial \tau}, \quad -\infty < \zeta < -1, \; \tau > 0, \tag{18}$$

$$T_s^*(0,\tau) = T_t^*(0,\tau), \quad \tau > 0, \tag{19}$$

$$\left.\frac{\partial T_s^*}{\partial \zeta}\right|_{\zeta=0-} - K_t^* \left.\frac{\partial T_t^*}{\partial \zeta}\right|_{\zeta=0+} = 1, \quad \tau > 0, \tag{20}$$

$$T_s^*(-1,\tau) = T_f^*(-1,\tau), \quad \tau > 0, \tag{21}$$

$$\left.\frac{\partial T_s^*}{\partial \zeta}\right|_{\zeta=-1+} = K_f^* \left.\frac{\partial T_f^*}{\partial \zeta}\right|_{\zeta=-1-}, \quad \tau > 0, \tag{22}$$

$$T_t^*(\zeta,\tau) \to 0, \quad \zeta \to \infty, \; \tau > 0, \tag{23}$$

$$T_f^*(\zeta,\tau) \to 0, \quad \zeta \to -\infty, \; \tau > 0, \tag{24}$$

$$T_t^*(\zeta,0) = 0, \quad 0 \leqslant \zeta < \infty, \tag{25}$$

$$T_s^*(\zeta,0) = 0, \quad -1 \leqslant \zeta \leqslant 0, \tag{26}$$

$$T_f^*(\zeta,0) = 0, \quad -\infty < \zeta \leqslant -1. \tag{27}$$

2.1. Solution of the problem

We perform the Laplace integral transform [17]

$$\overline{T}_{t,s,f}^*(\zeta,p) = \int_0^\infty T_{t,s,f}^*(\zeta,\tau) e^{-p\tau} d\tau. \tag{28}$$

on the heat conduction Eqs. (16)–(18) and the boundary conditions (19)–(24) with the homogeneous initial conditions (25)–(27) for the temperature. Thus we have

$$\frac{d^2 \overline{T}_t^*(\zeta,p)}{d\zeta^2} - \frac{p}{k_t^*}\overline{T}_t^*(\zeta,p) = 0, \quad 0 < \zeta < \infty, \tag{29}$$

$$\frac{d^2 \overline{T}_s^*(\zeta,p)}{d\zeta^2} - p\overline{T}_s^*(\zeta,p) = 0, \quad -1 < \zeta < 0, \tag{30}$$

$$\frac{d^2 \overline{T}_f^*(\zeta,p)}{d\zeta^2} - \frac{p}{k_f^*}\overline{T}_f^*(\zeta,p) = 0, \quad -\infty < \zeta < -1, \tag{31}$$

$$\overline{T}_s^*(0,p) = \overline{T}_t^*(0,p), \tag{32}$$

$$\left.\frac{d\overline{T}_s^*(\zeta,p)}{d\zeta}\right|_{\zeta=0-} - K_t^* \left.\frac{d\overline{T}_t^*(\zeta,p)}{d\zeta}\right|_{\zeta=0+} = \frac{1}{p}, \tag{33}$$

$$\overline{T}_s^*(-1,p) = \overline{T}_f^*(-1,p), \tag{34}$$

$$\left.\frac{d\overline{T}_s^*(\zeta,p)}{d\zeta}\right|_{\zeta=-1+} = K_f^* \left.\frac{d\overline{T}_f^*(\zeta,p)}{d\zeta}\right|_{\zeta=-1-}, \tag{35}$$

$$\overline{T}_t^*(\zeta,p) \to 0, \quad \zeta \to \infty, \tag{36}$$

$$\overline{T}_f^*(\zeta,p) \to 0, \quad \zeta \to -\infty. \tag{37}$$

The general solutions of the ordinary differential Eqs. (29)–(31) have the form:

$$\overline{T}_t^*(\zeta,p) = C_t(p) \exp\left(\zeta\sqrt{\frac{p}{k_t^*}}\right) + D_t(p) \exp\left(-\zeta\sqrt{\frac{p}{k_t^*}}\right), \quad 0 < \zeta < \infty, \tag{38}$$

$$\overline{T}_s^*(\zeta,p) = C_s(p) sh(\zeta\sqrt{p}) + D_s(p) ch(\zeta\sqrt{p}), \quad -1 < \zeta < 0, \tag{39}$$

$$\overline{T}_f^*(\zeta,p) = C_f(p) \exp\left(\zeta\sqrt{\frac{p}{k_f^*}}\right) + D_f(p) \exp\left(-\zeta\sqrt{\frac{p}{k_f^*}}\right), \quad -\infty < \zeta < -1, \tag{40}$$

where $C_{t,s,f}$ and $D_{t,s,f}$ are unknown functions of Laplace transformation parameter p. Satisfying the boundary condition (32) and the conditions of regularities (36) and (37) we find $D_t(p) = D_s(p)$, $C_t(p) = D_f(p) = 0$. Satisfying the remained boundary conditions (33)–(35), we obtain the Laplace transforms of temperatures in the following form:

$$\overline{T}_t^*(\zeta,p) = \frac{[\varepsilon_f sh(\sqrt{p}) + ch(\sqrt{p})]}{p\sqrt{p}\Delta(p)} \exp\left(-\zeta\sqrt{\frac{p}{k_t^*}}\right), \quad 0 \leqslant \zeta < \infty, \tag{41}$$

$$\overline{T}_s^*(\zeta,p) = \frac{\varepsilon_f sh[(1+\zeta)\sqrt{p}] + ch[(1+\zeta)\sqrt{p}]}{p\sqrt{p}\Delta(p)}, \quad -1 \leqslant \zeta \leqslant 0, \tag{42}$$

$$\overline{T}_f^*(\zeta,p) = \frac{1}{p\sqrt{p}\Delta(p)} \exp\left[(1+\zeta)\sqrt{\frac{p}{k_f^*}}\right], \quad -\infty < \zeta \leqslant -1, \tag{43}$$

where

$$\Delta(p) = (1+\varepsilon_t\varepsilon_f)\mathrm{sh}(\sqrt{p}) + (\varepsilon_t + \varepsilon_f)\mathrm{ch}(\sqrt{p}), \tag{44}$$

$$\varepsilon_t \equiv \frac{K_t^*}{\sqrt{k_t^*}} = \frac{K_t}{K_s}\sqrt{\frac{k_s}{k_t}}, \quad \varepsilon_f \equiv \frac{K_f^*}{\sqrt{k_f^*}} = \frac{K_f}{K_s}\sqrt{\frac{k_s}{k_f}}. \tag{45}$$

The dimensionless quantities $0 < \varepsilon_{t,f} < \infty$ (45) are known under the name "coefficient of thermal activity" [18], where ε_t characterizes thermal activity of the material of the top semi-space relative to the material of the strip, and ε_f – of the substrate to the strip.

We have introduced the notations

$$\lambda_t = \frac{1-\varepsilon_t}{1+\varepsilon_t}, \quad \lambda_f = \frac{1-\varepsilon_f}{1+\varepsilon_f}. \tag{46}$$

It is obvious that $-1 < \lambda_{t,f} < 1$ and if materials of all elements are the same, then $\lambda_t = \lambda_f = 0$. Taking into account relations (45) and (46) the denominator (44) can now written as

$$\Delta(p) = \frac{1}{2}(1+\varepsilon_t)(1+\varepsilon_f)\exp(\sqrt{p})[1 - \lambda\exp(-2\sqrt{p})], \tag{47}$$

where $\lambda = \lambda_t\lambda_f$. Representing the function $[1 - \lambda\exp(-2\sqrt{p})]^{-1}$ by the geometric series

$$\frac{1}{1-\lambda\exp(-2\sqrt{p})} = \sum_{n=0}^{\infty} \Lambda^n \exp(-2n\sqrt{p}), \tag{48}$$

where

$$\Lambda^n = \begin{cases} \lambda^n, & 0 \leqslant \lambda < 1, \\ (-1)^n|\lambda|^n, & -1 < \lambda \leqslant 0, \end{cases} \tag{49}$$

from Eq. (47) we obtain

$$\frac{1}{\Delta(p)} = \frac{2\exp(-\sqrt{p})}{(1+\varepsilon_t)(1+\varepsilon_f)} \sum_{n=0}^{\infty} \Lambda^n \exp(-2n\sqrt{p}). \tag{50}$$

Introducing the expression (50) into solutions (41)–(43) we find the transforms of the temperature

$$\overline{T}_t^*(\zeta,p) = \frac{1}{(1+\varepsilon_t)p\sqrt{p}} \sum_{n=0}^{\infty} \Lambda^n \left\{ \exp\left[-\left(2n + \frac{\zeta}{\sqrt{k_t^*}}\right)\sqrt{p}\right] + \lambda_f \exp\left[-\left(2n+2+\frac{\zeta}{\sqrt{k_t^*}}\right)\sqrt{p}\right]\right\}, \quad 0 \leqslant \zeta < \infty, \tag{51}$$

$$\overline{T}_s^*(\zeta,p) = \frac{1}{(1+\varepsilon_t)p\sqrt{p}} \sum_{n=0}^{\infty} \Lambda^n \{\exp[-(2n-\zeta)\sqrt{p}] + \lambda_f \exp[-(2n+2+\zeta)\sqrt{p}]\}, \quad -1 \leqslant \zeta \leqslant 0, \tag{52}$$

$$\overline{T}_f^*(\zeta,p) = \frac{2}{(1+\varepsilon_t)(1+\varepsilon_f)p\sqrt{p}} \sum_{n=0}^{\infty} \Lambda^n \exp\left\{-\left[2n+1-\frac{(1+\zeta)}{\sqrt{k_f^*}}\right]\sqrt{p}\right\}, \quad -\infty < \zeta \leqslant -1. \tag{53}$$

The transforms (51)–(54) are inverted by the inversion formula [19]

$$L^{-1}\left[\frac{\exp(-\sqrt{ap})}{p\sqrt{p}};\tau\right] = 2\sqrt{\tau}\,\mathrm{ierfc}\left(\frac{1}{2}\sqrt{\frac{a}{\tau}}\right), \quad a>0, \tag{54}$$

where

$$\mathrm{ierfc}(x) = \frac{\exp(-x^2)}{\sqrt{\pi}} - x\,\mathrm{erfc}(x). \tag{55}$$

As a result we find the temperatures in the closed form:

$$T_t^*(\zeta,\tau)=\frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\sum_{n=0}^{\infty}\Lambda^n T_{t,n}^*(\zeta,\tau),\ 0\leqslant\zeta<\infty,\ \tau\geqslant0, \tag{56}$$

$$T_{t,n}^* = \mathrm{ierfc}\left[\left(2n+\frac{\zeta}{\sqrt{k_t^*}}\right)\frac{1}{2\sqrt{\tau}}\right] + \lambda_f\,\mathrm{ierfc}\left[\left(2n+2+\frac{\zeta}{\sqrt{k_t^*}}\right)\frac{1}{2\sqrt{\tau}}\right],\ n=0,1,2\ldots, \tag{57}$$

$$T_s^*(\zeta,\tau)=\frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\sum_{n=0}^{\infty}\Lambda^n T_{s,n}^*(\zeta,\tau),\ -1\leqslant\zeta\leqslant0,\ \tau\geqslant0, \tag{58}$$

$$T_{s,n}^*(\zeta,\tau)=\mathrm{ierfc}\left(\frac{2n-\zeta}{2\sqrt{\tau}}\right)+\lambda_f\,\mathrm{ierfc}\left(\frac{2n+2+\zeta}{2\sqrt{\tau}}\right),\ n=0,1,2\ldots, \tag{59}$$

$$T_f^*(\zeta,\tau)=\frac{4\sqrt{\tau}}{(1+\varepsilon_t)(1+\varepsilon_f)}\sum_{n=0}^{\infty}\Lambda^n T_{f,n}^*(\zeta,\tau),\ -\infty<\zeta\leqslant-1,\ \tau\geqslant0, \tag{60}$$

$$T_{f,n}^*(\zeta,\tau)=\mathrm{ierfc}\left[\left(2n+1-\frac{1+\zeta}{\sqrt{k_f^*}}\right)\frac{1}{2\sqrt{\tau}}\right],\ n=0,1,2\ldots. \tag{61}$$

Let us define the intensity of the heat fluxes in the top semi-space, the strip and in the substrate as:

$$q_t(z,t) \equiv -K_t\frac{\partial T_t(z,t)}{\partial z}, \quad 0\leqslant z\leqslant\infty, t\geqslant0, \tag{62}$$

$$q_s(z,t) \equiv K_s\frac{\partial T_s(z,t)}{\partial z}, \quad -1\leqslant z\leqslant0, t\geqslant0, \tag{63}$$

$$q_f(z,t) \equiv K_f\frac{\partial T_f(z,t)}{\partial z}, \quad -\infty\leqslant z\leqslant-1, t\geqslant0, \tag{64}$$

or, taking denotes (14) and (15) into account, their dimensionless values:

$$q_t^*(\xi,\tau) \equiv \frac{q_t(z,t)}{q} = -K_t^*\frac{\partial T_t^*(\zeta,\tau)}{\partial\zeta}, \quad 0\leqslant\zeta\leqslant\infty,\ \tau\geqslant0, \tag{65}$$

$$q_s^*(\zeta,\tau) \equiv \frac{q_s(z,t)}{q} = \frac{\partial T_s^*(\zeta,\tau)}{\partial\zeta}, \quad -1\leqslant\zeta\leqslant0,\ \tau\geqslant0, \tag{66}$$

$$q_f^*(\zeta,\tau) \equiv \frac{q_f(z,t)}{q} = K_f^*\frac{\partial T_f^*(\zeta,\tau)}{\partial\zeta}, \quad -\infty<\zeta\leqslant-1,\ \tau\geqslant0. \tag{67}$$

Taking into account that d[ierfc(x)]/dx = −erfc(x), after differentiating the dimensionless temperatures (56)–(61) with respect to ζ from the relations (65)–(67) we obtain:

$$q_t^*(\zeta,\tau) = \frac{\varepsilon_t}{(1+\varepsilon_t)}\sum_{n=0}^{\infty}\Lambda^n q_{t,n}^*(\zeta,\tau), \quad 0\leqslant\zeta<\infty, \quad \tau\geqslant0, \tag{68}$$

$$q_{t,n}^*(\zeta,\tau) = \mathrm{erfc}\left[\left(2n+\frac{\zeta}{\sqrt{k_t^*}}\right)\frac{1}{2\sqrt{\tau}}\right] + \lambda_f\,\mathrm{erfc}\left[\left(2n+2+\frac{\zeta}{\sqrt{k_t^*}}\right)\frac{1}{2\sqrt{\tau}}\right], \quad n=0,1,2,\ldots, \tag{69}$$

$$q_s^*(\zeta,\tau) = \frac{1}{(1+\varepsilon_t)}\sum_{n=0}^{\infty}\Lambda^n q_{s,n}^*(\zeta,\tau), \quad -1\leqslant\zeta\leqslant0, \quad \tau\geqslant0, \tag{70}$$

$$q_{s,n}^*(\zeta,\tau) = \mathrm{erfc}\left(\frac{2n-\zeta}{2\sqrt{\tau}}\right) - \lambda_f\,\mathrm{erfc}\left(\frac{2n+2+\zeta}{2\sqrt{\tau}}\right), \quad n=0,1,2\ldots \tag{71}$$

$$q_f^*(\zeta,\tau)=\frac{2\varepsilon_f}{(1+\varepsilon_t)(1+\varepsilon_f)}\sum_{n=0}^{\infty}\Lambda^n q_{f,n}^*(\zeta,\tau),\ -\infty<\zeta\leqslant-1,\ \tau\geqslant0, \tag{72}$$

$$q_{f,n}^*(\zeta,\tau)=\mathrm{erfc}\left[\left(2n+1-\frac{(1+\zeta)}{\sqrt{k_f^*}}\right)\frac{1}{2\sqrt{\tau}}\right],\ n=0,1,2\ldots. \tag{73}$$

2.2. Some particular solutions of the problem

The maximal temperature is reached on a plane of friction $\zeta = 0$ and as follows from the formulae (56)–(59), is equal:

$$T^*(\tau) \equiv T_t^*(0,\tau) = T_s^*(0,\tau) = \frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\sum_{n=0}^{\infty}\Lambda^n T_n^*(\tau), \quad \tau\geqslant0, \tag{74}$$

$$T_n^*(\tau) \equiv T_{t,n}^*(0,\tau) = T_{s,n}^*(0,\tau) = \mathrm{ierfc}\left(\frac{n}{\sqrt{\tau}}\right) + \lambda_f\,\mathrm{ierfc}\left(\frac{n+1}{\sqrt{\tau}}\right), \quad n=0,1,2\ldots, \tag{75}$$

The corresponding dimensionless intensities of heat fluxes on a surface $\zeta = 0$ it are found from formulae (68)–(71) as:

$$q_t^*(0,\tau) = \frac{\varepsilon_t}{(1+\varepsilon_t)}\sum_{n=0}^{\infty}\Lambda^n q_{t,n}^*(0,\tau), \quad \tau\geqslant0, \tag{76}$$

$$q_{t,n}^*(0,\tau) = \mathrm{erfc}\left(\frac{n}{\sqrt{\tau}}\right) + \lambda_f\,\mathrm{erfc}\left(\frac{n+1}{\sqrt{\tau}}\right), \quad n=0,1,2\ldots, \tag{77}$$

$$q_s^*(0,\tau) = \frac{1}{(1+\varepsilon_t)}\sum_{n=0}^{\infty}\Lambda^n q_{s,n}^*(0,\tau), \quad \tau\geqslant0, \tag{78}$$

$$q_{s,n}^*(0,\tau) = \mathrm{erfc}\left(\frac{n}{\sqrt{\tau}}\right) - \lambda_f\,\mathrm{erfc}\left(\frac{n+1}{\sqrt{\tau}}\right), \quad n=0,1,2\ldots. \tag{79}$$

From the relations (76)–(79) it follows that

$$\begin{aligned} q_t^*(0,\tau)+q_s^*(0,\tau) &= \tfrac{1}{(1+\varepsilon_t)}\sum_{n=0}^{\infty}\Lambda^n\left[(\varepsilon_t+1)\mathrm{erfc}\left(\tfrac{n}{\sqrt{\tau}}\right)+\lambda_f(\varepsilon_t-1)\mathrm{erfc}\left(\tfrac{n+1}{\sqrt{\tau}}\right)\right]= \\ &= \sum_{n=0}^{\infty}\Lambda^n\left[\mathrm{erfc}\left(\tfrac{n}{\sqrt{\tau}}\right)-\lambda_f\lambda_t\,\mathrm{erfc}\left(\tfrac{n+1}{\sqrt{\tau}}\right)\right]=\sum_{n=0}^{\infty}\left[\Lambda^n\mathrm{erfc}\left(\tfrac{n}{\sqrt{\tau}}\right)-\Lambda^{n+1}\mathrm{erfc}\left(\tfrac{n+1}{\sqrt{\tau}}\right)\right]= \\ &= \mathrm{erfc}(0)=1, \end{aligned} \tag{80}$$

i.e. the boundary condition (20) is satisfied.

In the case of identical physical properties of a strip and substrate ($K_s = K_f$, $k_s = k_f$) from formulae (14), (45) and (46) it follows that $K_f^* = 1, k_f^* = 1, \varepsilon_f = 1, \lambda_f = 0, \Lambda = 0$. The Eqs. (56)–(61), (68)–(73) at $n = 0$ give the solution of the thermal problem of friction for two homogeneous semi-spaces [20]:

$$T_t^*(\zeta,\tau) = \frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\mathrm{ierfc}\left(\frac{\zeta}{2\sqrt{k_t^*\tau}}\right), \quad 0\leqslant\zeta<\infty,\ \tau\geqslant0, \tag{81}$$

$$T_f^*(\zeta,\tau) = \frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\mathrm{ierfc}\left(-\frac{\zeta}{2\sqrt{\tau}}\right), \quad -\infty<\zeta\leqslant0,\ \tau\geqslant0, \tag{82}$$

$$q_t^*(\zeta,\tau) = \frac{\varepsilon_t}{(1+\varepsilon_t)}\mathrm{erfc}\left(\frac{\zeta}{2\sqrt{k_t^*\tau}}\right), \quad 0\leqslant\zeta<\infty,\ \tau\geqslant0, \tag{83}$$

$$q_f^*(\zeta,\tau) = \frac{1}{(1+\varepsilon_t)}\mathrm{erfc}\left(-\frac{\zeta}{2\sqrt{\tau}}\right), \quad -\infty<\zeta\leqslant0,\ \tau\geqslant0. \tag{84}$$

At identical material properties of the top semi-space and strips ($K_t = K_s$, $k_t = k_s$) from the formulae (14), (45) and (46) it follows that $K_t^* = 1, k_t^* = 1, \varepsilon_t = 1, \lambda_t = 0, \Lambda = 0$ and we obtain

$$T_t^* = \sqrt{\tau}\left[\mathrm{ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)+\lambda_f\,\mathrm{ierfc}\left(\frac{2+\zeta}{2\sqrt{\tau}}\right)\right],\ 0\leqslant\zeta<\infty,\ \tau\geqslant0, \tag{85}$$

$$T_s^*(\zeta,\tau) = \sqrt{\tau}\left[\mathrm{ierfc}\left(\frac{-\zeta}{2\sqrt{\tau}}\right)+\lambda_f\,\mathrm{ierfc}\left(\frac{2+\zeta}{2\sqrt{\tau}}\right)\right],\ -1\leqslant\zeta\leqslant0,\ \tau\geqslant0, \tag{86}$$

$$T_f^*(\zeta,\tau)=\frac{2\sqrt{\tau}}{(1+\varepsilon_f)}\text{ierfc}\left[\left(1-\frac{1+\zeta}{\sqrt{k_f^*}}\right)\frac{1}{2\sqrt{\tau}}\right],\ -\infty<\zeta\leqslant-1,\ \tau\geqslant0,\quad(87)$$

$$q_t^*(\zeta,\tau)=\frac{1}{2}\left[\text{erfc}\left(\frac{\zeta}{\sqrt{2\tau}}\right)+\lambda_f\text{erfc}\left(\frac{2+\zeta}{2\sqrt{\tau}}\right)\right],\ 0\leqslant\zeta<\infty,\ \tau\geqslant0,\quad(88)$$

$$q_s^*(\zeta,\tau)=\frac{1}{2}\left[\text{erfc}\left(\frac{-\zeta}{\sqrt{2\tau}}\right)-\lambda_f\text{erfc}\left(\frac{2+\zeta}{2\sqrt{\tau}}\right)\right],\ -1\leqslant\zeta\leqslant0,\ \tau\geqslant0,\quad(89)$$

$$q_f^*(\zeta,\tau)=\frac{\varepsilon_f}{(1+\varepsilon_f)}\text{erfc}\left[\left(1-\frac{(1+\zeta)}{\sqrt{k_f^*}}\right)\frac{1}{2\sqrt{\tau}}\right],\ -\infty<\zeta\leqslant-1,\ \tau\geqslant0.\quad(90)$$

In the case of identical physical properties of the top semi-space and substrate ($K_t = K_f = K,\ k_t = k_f = k$), the dimensionless temperatures and heat fluxes it is calculated under formulae (56)–(61), (68)–(73), believing in them $\varepsilon_t = \varepsilon_f \equiv \varepsilon,\ \lambda_t = \lambda_f = \lambda,\ \Lambda = \lambda^2$, where, taking the notations (14), (45) and (46) into account,

$$K^* = \frac{K}{K_s},\quad k^* = \frac{k}{k_s},\quad \varepsilon = \frac{K^*}{\sqrt{k^*}},\quad \lambda = \frac{1-\varepsilon}{1+\varepsilon}.\quad(91)$$

If the materials of all elements are identical ($K_t = K_s = K_f$, $k_t = k_s = k_f$) then in the Eqs. (81)–(84) it is necessary to put additively $k_t^* = 1, \varepsilon_t = 1$ and $k_f^* = 1, \varepsilon_f = 1, \lambda_f = 0$ in the Eqs. (85)–(90).

2.3. Numerical analysis

The input parameters of a problem are the spatial coordinate z, the time t, the ratios of heat conduction $K_{t,s,f}$ and diffusivity $k_{t,s,f}$, the thickness of the strip d, the coefficient of friction f, the sliding speed V and pressure p_0. All calculations for temperatures and heat fluxes were executed for fixed values $f = 0.3, V = 5\ \text{ms}^{-1}$, $p_0 = 1$ MPa in the case, when the properties of materials top and bottom of semi-space are identical($K_t = K_f = K,\ k_t = k_f = k$) and made of St 40H steel: $K = 41.9\text{W(mk)}^{-1}$, $k = 1.2 \times 10^{-5}\ \text{m}^2\ \text{s}^{-1}$. The strip materials are aluminum Al: $K_s = 209$ W $(\text{mk})^{-1}$, $k_s = 8.6 \times 10^{-5}\ \text{m}^2\ \text{s}^{-1}$ or zirconium dioxide ZrO_2: $K = 2$ W $(\text{mk})^{-1}$, $k = 0.08 \times 10^{-5}\ \text{m}^2\ \text{s}^{-1}$. The initial temperature equals $T_0 = 20$ °C. The solid curves in all figures correspond to results of calculations for the aluminum strip, the dashed curves correspond to the zirconium dioxide strip.

The evolution of temperatures on the contact surface ($z = 0$) and on the interface between the strip and substrate ($z = -d$), is shown in Fig. 2. It is observed that for fixed values of strip thickness ($d = 300$ μm), the temperature on friction surface for steel semi-space–ceramic strip tribosystem is significantly higher than contact temperature for steel semi-space–aluminum strip tribosystem during whole period of sliding. For the same strip thickness the opposite effect is observed on the interface $z = -d$ – the temperature during friction between steel and aluminum strip is higher, than in case of the ceramic strip.

The temperature distribution in the elements of tribosystem on depth $|z|$ from a plane of friction $z = 0$ is shown in Fig. 3. The maximal temperature is reached on a surface of friction and in case of the zirconium dioxide strip equals $T = 148$ °C whereas for the aluminum strip reaches $T = 110$ °C. In aluminium strip the temperature is nearly constant on the whole depth, whereas in ceramic one decreases linearly with distance from the friction surface. The temperature in the steel substrate reaches significant values only in case of the aluminium strip. Some other character has temperature distribution on depth in the top semi-space. In case of the ceramics strip the temperature in the top semi-space is always higher than temperature in the case of aluminum based strip. In substrate and in the top semi-space both, temperature reaches

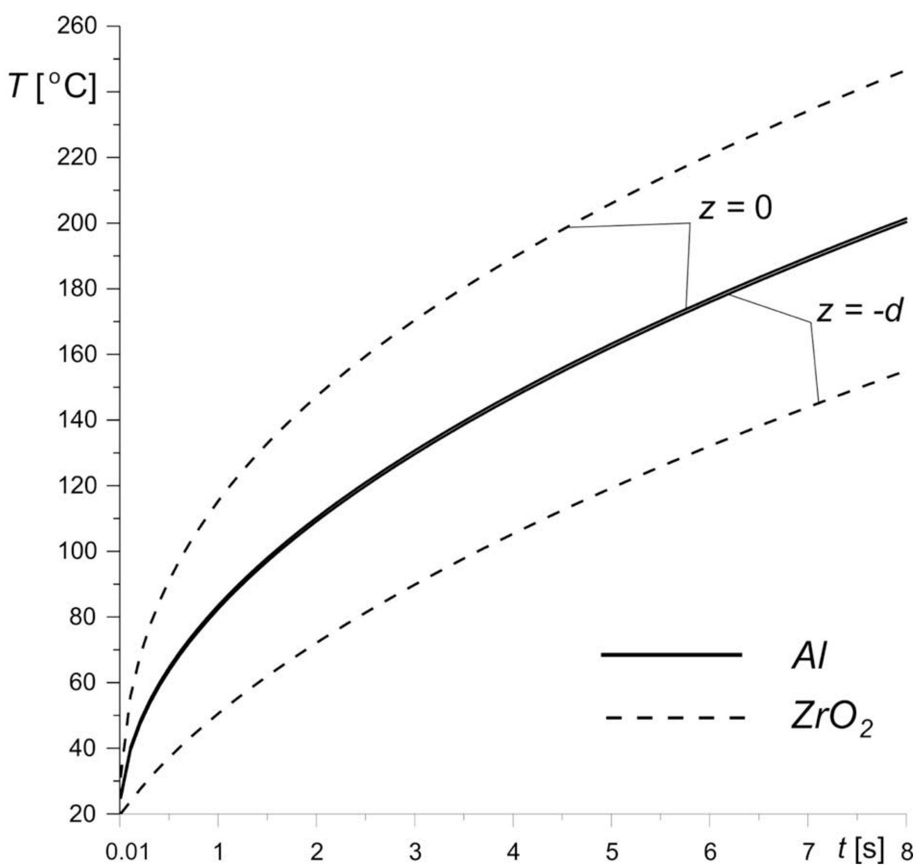

Fig. 2. Evolution of the temperature T for St–Al–St (solid curves) and St–ZrO_2–St (dashed curves) tribosystems for two values of the distance $|z|$ from surface of friction at the strip thickness $d = 300$ μm.

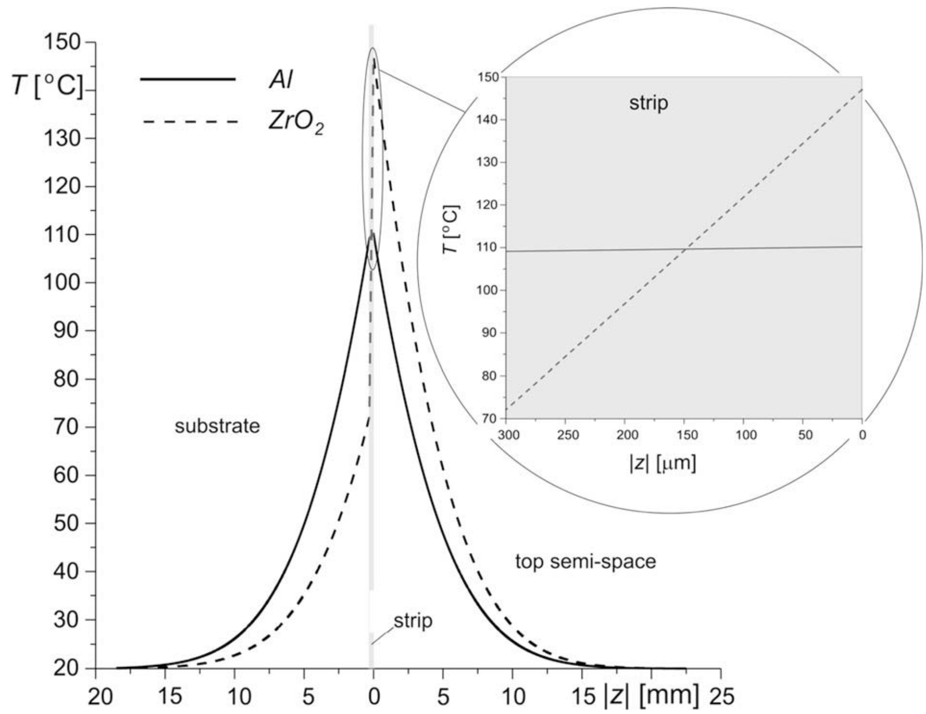

Fig. 3. Dependence of the temperature T for St–Al–St (solid curves) and St–ZrO_2–St (dashed curves) tribosystems on the distance $|z|$ from surface of friction for values of time $t = 2$ s and the strip thickness $d = 300$ μm.

its initial value $T_0 = 20$ °C at the depth of 15 mm, which correspond to 50 strips of $d = 300$ μm thickness.

For fixed time moment $t = 2$ s, thickness of ceramic strip should be equal 5 mm in order to achieve temperatures range in nonlinear

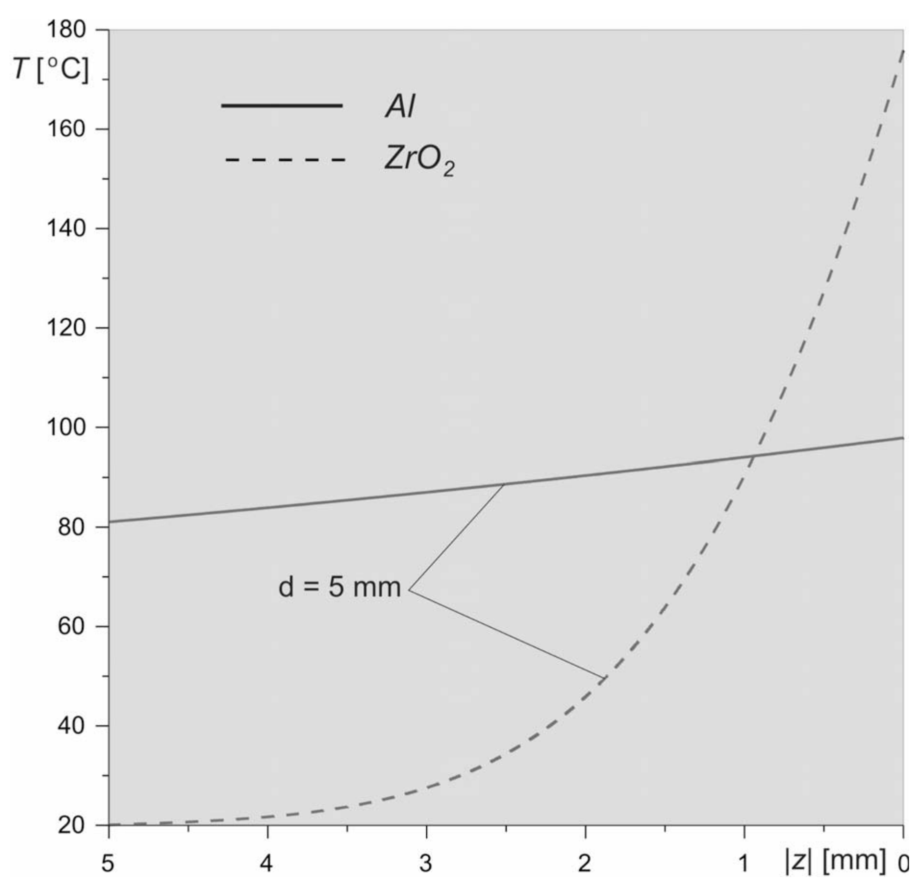

Fig. 4. Distribution of the temperature T in the aluminum (solid line) and zirconium dioxide (dashed curve) for values of time $t = 2$ s and the strip thickness $d = 5$ mm.

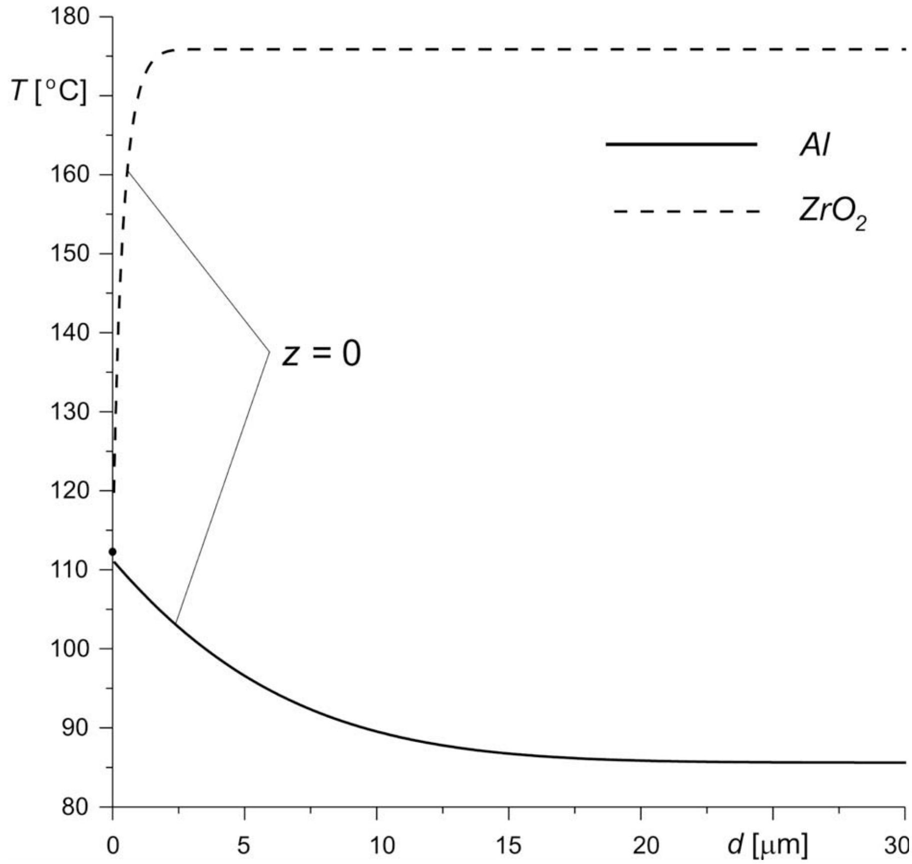

Fig. 5. Dependence of the contact temperature T on the strip thickness d for St–Al–St (solid curve) and St–ZrO_2–St (dashed curve) at time moment $t = 2$ s.

decrease from maximal value of $T_0 = 175$ °C on friction surface $z = 0$ to the initial value of $T_0 = 20$ °C on interface surfaces $z = -d$, see Fig. 4.

At the same time, the adequate decreasing of temperature in aluminum strip is insignificant and has linear character. This difference between both cases for temperature is caused by the fact that heat is conducted deeper in strip made of material with higher thermal diffusivity coefficient such as aluminium.

Dependence of temperature on a surface of friction $z = 0$ from the strip thickness d for the fixed time moment $t = 2$ s is shown in Fig. 5. At $d = 0$ we have the solution of the problem in the case of identical physical properties of a semi-space, strip and substrate (the formulae (81)–(84) at $\varepsilon_t = 1, k_t^* = 1$). For defined in the beginning parameters, the final temperature equals $T = 112$ °C. Aluminum has higher thermal diffusivity coefficient than steel. Therefore, with the increase of strip thickness, better heat-conducting materials such as aluminum will replace worse heat-conducting materials (steel) in order to cause decrease of temperature on the friction surface. It can be observed in Fig. 5 that increasing of aluminum strip thickness causes in beginning the decrease of temperature on the friction surface, to become afterwards from thickness qual $d \cong 20$ μm, on the constant level $T \cong 85$ °C. The temperature evolution for ceramic strip ZrO_2 is observed on the contrary way. As ZrO_2 dioxide is worse heat-conducting materials than steel, so its thermal resistance is much higher [21]. As a result, with increase of strip thickness the temperature on the friction surface increases, too. It is shown in Fig. 5 that increase of strip thickness causes immediately increase of temperature from $T = 112$ °C for $d = 0$ to $T = 175$ °C for $d \cong 2$ μm. Subsequent increase of strip thickness does not change the maximal temperature.

Dimensionless heat fluxes intensities directed to each elements of tribosystem are defined by formulas (62)–(64) in the form of adequate heat flux intensities and constant value of friction power $q = fVp_0$ ratio. That is why, the numerical analysis was carried out only for dimensionless heat fluxes.

The evolution of the dimensionless heat fluxes intensities directed in the top semi-space (q_t^*, $\zeta \geqslant 0$) and in the strip (q_s^*, $-1 \leqslant \zeta \leqslant 0$) is shown in Fig. 6. It can be noticed that the aluminum strip absorbs more heat than others (Fig. 6a) and that ceramic one absorbs much less heat (Fig. 6b) than steel top semi-space. The maximal dimensionless heat fluxes intensity $q_{t,s}^*$ takes place on a plane of friction $\zeta = 0$ and decreases with the distance from it. On the contrary to the temperature, the heat fluxes intensity increases in time and reaches the steady state. The quickest stationary state is reached on a plane of friction $\zeta = 0$. It is clearly shown that the sum of dimensionless heat fluxes intensities on this plane, directed in the top semi-space and a strip, always equals one, thus the boundary condition (20) is satisfied.

3. Conclusions

The analytical solution of a heat conduction problem of friction for tribosystem, consisting of semi-space, sliding with constant speed on a surface of the strip deposited on the substrate, was obtained. Distribution of temperatures and heat fluxes intensities in elements of friction pair: steel–aluminum–steel and steel–zirconium dioxide–steel, was investigated. It was found, that the maximal temperature takes place on a surface of friction. During whole period of sliding, the contact temperature of the ceramics strip is significantly higher than temperature of a aluminum based strip. The temperature in the steel top semi-space decreases with distance from a surface of friction much quicker in case of a strip from zirconium dioxide, than in the case of aluminum strip. The steel substrate heats up to a significant level only in case of the ceramics

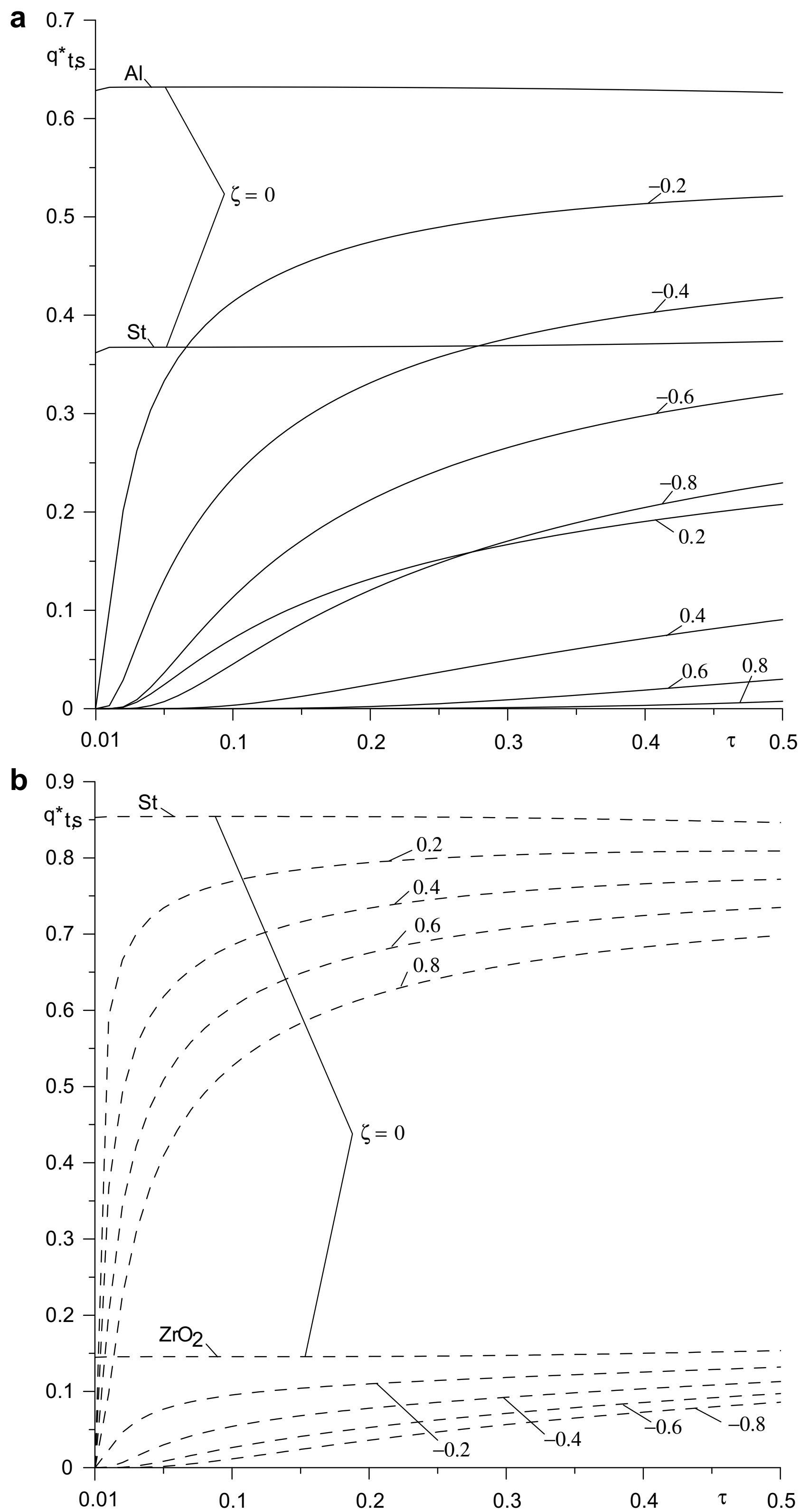

Fig. 6. Evolution of the dimensionless heat fluxes q^*: (a) for St–Al–St tribosystem (solid curves) and (b) for St–ZrO_2–St tribosystem (dashed curves) for several values of the dimensionless distance ζ from surface of friction.

strip. The maximal heat fluxes intensity takes place on a plane of friction and decreases with the distance from it.

We note that the increase of temperature on a surface of friction causes rapid wear [22,23]. Simultaneous influence of frictional heating and wear on the temperature distribution in two sliding semi-spaces has been investigated in article [12]. For the tribosystem being in this paper under study, the same problem will be researched in the nearly future.

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